THE CONVECTIVE STABILITY OF A FLUID IN A HOLLOW CUBE

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The experimental model is shown in Fig. 1. Hollow cube 2 with a volume of 1 cm^3 was formed inside a plexiglas assembly 74 x 74 x 16 mm in size. The block was compressed between brass plates (4 and 5) 144 x 144 x 6 mm in size. Plate 4 was heated by an electric heater, and plate 5 was cooled by a spray-type cooler. The hollow cube was filled with fluid by means of channels 8, which enter at the cube edges. The motion of this fluid was made visible by suspending light-scattering particles of aluminum powder in it. The experiments were performed with water. Thermal contact between the plates and the block was improved with machine oil, and between the [plexiglas] plates in the assembly with water bled from the hollow cube.

Copper-constantan thermocouples with five junctions measured the temperature differences ΔT_1, ΔT_2, and ΔT_3 between, respectively, the boundaries of plate 1, hollow cube 2, and plate 3 along the axis of symmetry, as well as the temperature difference ΔT_0 between junctions 6 and 7. Junction 7 was in contact with the bulb of a thermometer showing the room temperature T_R. The temperature $T_R + \Delta T_0 - (\Delta T_2/2)$ at the center of the hollow cube was selected equal to T_R, and this condition was satisfied for $\Delta T_0 = \Delta T_2/2$ and T_R = = const. After switching off the heater and the cooler, the steady-state temperature difference $\Delta T_1 + \Delta T_2 + \Delta T_3$ between them decreased exponentially, but the experimental constant $\Delta T_1/\Delta T_3$ and the temperature at the cube center did not change significantly. The fact that the results for this case and for steady-state regimes are in close agreement enables us to treat this case as if it were quasi steady.

The Rayleigh and Nusselt numbers were given by:

$$R = g\frac{\beta}{\nu\chi}\, l^3\Delta T_2, \qquad N = \frac{\varkappa_1{}^*\Delta T_1 + \varkappa_3{}^*\Delta T_3}{\varkappa\Delta T_2}\,.$$

Here g is the acceleration of gravity, β, ν, χ, and $\varkappa$ are, respectively, the coefficients of volume expansion, of kinematic viscosity,

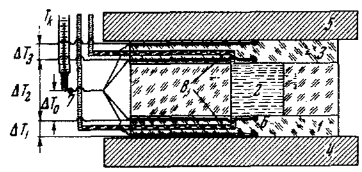

Fig. 1

and of thermal diffusivity and thermal conductivity for the water inside the hollow cube, l is the length of the cube edge. The experimental constants $\varkappa_1^*$ and $\varkappa_3$ are found, when the fluid is heated from above, from the following equations:

$$\varkappa_1{}^*\Delta T_1 = \varkappa\Delta T_2 = \varkappa_3{}^*\Delta T_3, \quad \text{i.e., for } N = 1.$$

In this case the fluid is in mechanical equilibrium and heat is transferred purely by means of heat conduction. When this equilibrium breaks down because of fluid passing through the hollow cube, i.e., in the case of convection, $\varkappa_1^*\Delta T_1 = \varkappa_3^*\Delta T_3 > \varkappa\Delta T_2$ and $N > 1$.

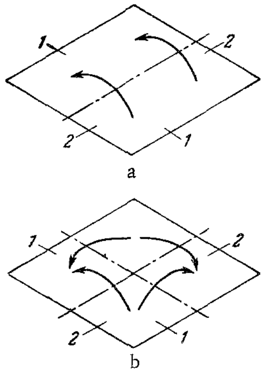

Fig. 2

Consequently, $N - 1 \geq 0$, equal to the ratio of the heat fluxes transported by the motion and the conduction of the fluid, can serve as a dimensionless measure of convective heat transfer, and as a measure of the flow rate.

When the fluid is heated from below, two forms of convective laminar flow are discovered. These two forms are shown diagrammatically in Fig. 2 as they appear at the central horizontal plane of the hollow cube. In case a (the first flow) the fluid circulates vertically, rising in one half of the hollow cube and descending in the other. In case b (the second flow) the fluid rises in two [diagonally] opposite vertical quarters of the cube and flows down through the other two. In both cases the fluid moves in approximately elliptical paths. The motion is more nearly elliptical with lower and lower flow rates. The axes of the ellipses are not quite vertical, being slightly inclined relative to the cube axes in the direction of fluid motion. Photographs of the first flow in vertical cross sections 1-1 and 2-2 (Fig. 2a) are reproduced in Figs. 3a and b, respectively ($R = 10.85 \cdot 10^3$, $N = 2.80$; the shutter speed was 0.5 x 0.5 sec, and the exposure took 15-30 sec). Photographs of the second flow in vertical cross section 1-1 (Fig. 2b) practically reproduce Fig. 3a in appearance, and in cross section 2-2 are its mirror image. Thus, the second flow has mirror-symmetry relative to the vertical diagonal planes. Consequently, the direction of fluid motion and the inclination of the ellipses on both sides of the central vertical plane to which they are parallel are the same for the first flow and opposite for the second. In the case of the second flow there are two such planes. In both types of flow the fluid is at rest at

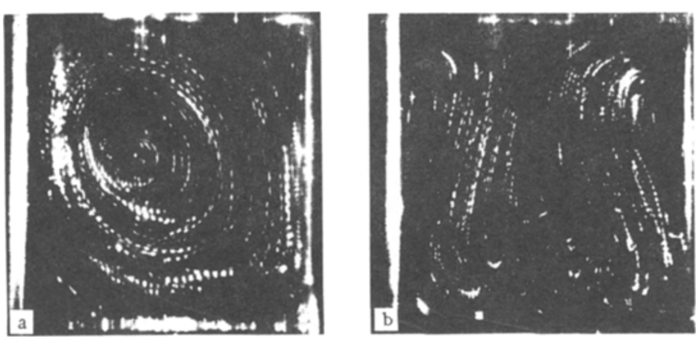

Fig. 3

the walls (because of adhesion) and at the center of the ellipses, so that the nodal planes coincide with the cube faces, and the nodal straight lines (the axial lines in Fig. 2) are situated in the central horizontal plane in the same way as for transverse oscillations of a square plate. Our inverting the model about horizontal axes can rotate the flow in the horizontal plane by $\pi/2$ or reverse its direction of motion.

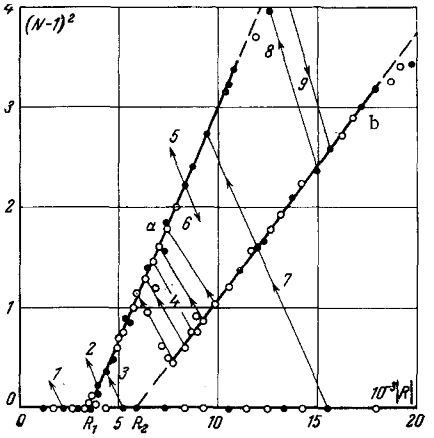

Fig. 4

The equations for heat transfer by the first and second flows are represented in Fig. 4 by the straight lines a and b, respectively. These lines intersect the axis of abscissas $(N-1)^2 = 0$ at points R_1 and R_2, which are the critical points. It was found that $R_1 = 3650 \pm 100$ and $R_2 = 6000 \pm 200$. The straight line a is described by the equation $(N-1)^2 = 0.449 \cdot 10^{-3} (R - R_1)$, and the straight line b by the equation $(N-1)^2 = 0.267 \cdot 10^{-3} (R - R_2)$. Thus, Landau's law [1-5] turns out to be valid for both types of motion. The numerical coefficient is connected with the critical number in both equations in the same way: $c_i R_i = 1.62 \pm 0.02$ $(i = 1, 2)$. For $R > 3R_i$ the experimental points deviate to the right from the straight lines a and b and lie on the straight lines if $(N-1)^3$ is plotted on the axis of ordinates (the dark circles denote the points obtained for steady-state regimes, and the open circles those for quasi-steady-state regimes).

As the heating decreased, the second flow was not completely damped, but for $R > 1.3R_2$ and for $N - 1 > 0.6$ it changed into the first, more stable flow (lines 4). During the change the elliptical motion in one half of the hollow cube stopped and reversed direction with a change of inclination of the ellipses. Change in the direction of motion by pumping fluid through the hollow cube or by deforming the temperature field in the assembly can cause the first flow to change into the less stable second flow (line 9), and vice-versa (line 8). Close to R_1 (in the experiments for $R < R_2$) the first flow cannot change into the second, and so after it is disturbed it always reestablishes itself (line 2). For $R < R_1$ the first flow is damped. Mechanical equilibrium of the fluid is reestablished after it is disturbed (line 1). Temperature and hydrodynamic perturbations are rapidly damped for $R < R_1$ and destroy the equilibrium for $R > R_1$.

Steady flow for a given temperature difference $\Delta T_1 + \Delta T_2 + \Delta T_3$ between the faces of the block can only be damped out by inverting the model so that the heater is on top. After inversion, the Archimedean forces damp the fluid motion in one second, and the perturbed temperature field reorganizes itself in several minutes. The transitions caused by inverting the model are shown by lines 3 and 7. After reinversion, the Archimedean forces destroy the equilibrium, and there is a flow from the heated face which perturbs the fluid in the hollow cube in a complicated manner. After several seconds the complicated flow is replaced by laminar flow, which assumes a steady-state character after several minutes. The first flow results in the case of transition 3, and either the second or first flow for transition 7. In these cases, as a result of the rising of hot fluid and the sinking of cold fluid, the upper cold face of the hollow cube is warmed, and the lower cold face cools, while the potential energy of the fluid in the gravity field decreases. Decrease in the temperature difference ΔT_2 between the cube faces causes increase in the temperature differences ΔT_1 and ΔT_3 between the faces of the [brass] plates covering the hollow cube. The insignificant decrease in the over-all difference of these temperatures is caused by some decrease in the total thermal resistance of the assembly due to convection in the hollow cube. Thus, increase of the heat flux through the plate and the hollow cube occurs during decrease of heat transfer by pure fluid heat conduction. It follows from this that for a given temperature difference between the assembly faces the change in heat transfer due to perturbed motion of the fluid is greater in absolute magnitude but of opposite sign to the change in heat transfer resulting from the thermal conductivity of the fluid (lines 1-9).

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